Bodies of Revolution at Angle of Attack in High Supersonic Flow

Ingeborg Ginzel*
Martin Marietta Corporation, Baltimore, Md.

Hayes[1] suggested that the pressure at station s_1 (measured along the meridian) of a body of revolution in axisymmetric flow be calculated by an improved Newton-Busemann equation

$$\frac{1}{-}C_{pb}(s_1) = (1-\epsilon)\sin^2\sigma(s_1) - \bar{K}\int_0^{s_1} q_t^2 dm$$

$$dm = (q_n/q_t)(y/y_1)ds \tag{1}$$

where $\bar{K}$ is the mean curvature of the streamlines in the shock layer

$$\bar{K} = |K| + \frac{\Delta}{2}K^2 + \frac{1}{\Delta}\int\frac{d^2z}{ds^2}dz \tag{2}$$

For the streamlines in the shock layer, he suggested hyperbolas,

$$\frac{dz}{ds} = \epsilon\frac{q_n}{q_t} + \frac{z}{\Delta}\left(\frac{d\Delta}{ds} - \epsilon\frac{q_n}{q_t}\right) \tag{3}$$

Here x is the coordinate of the shock meridian along its axis and y its coordinate perpendicular to the axis; z is measured from shock to body perpendicular to the shock and assumes the value Δ at the body; q_t and q_n are the velocities parallel and perpendicular to the shock, and ϵ is the density ratio $\epsilon = \rho_\infty/\rho$ across the shock.

In Ref. 2, Hayes suggested a constant-density approach to the angle-of-attack problem for a shock at an angle of attack α by generalizing in Eq. (1)

$$dm = (q_n/q_t)(ydw/y_1dw_1)ds \tag{4}$$

where, now, ydw is the component of the streamline binormal to the meridian. An attempt is made here to calculate the pressure in the plane of symmetry at the body of revolution, assuming the density ϵ and the velocities q_n and q_b (binormal) to be constant across the shock layer but varying with the varying shock angle σ. This means

$$\Delta(s_1) = \frac{\epsilon}{y_1}\int^{s_1} y\tan(\sigma\pm\alpha)ds\frac{dw}{dw_1} \tag{5a}$$

$$\int q_t^2 dm = \frac{1}{y_1}\int y\sin(\sigma\pm\alpha)\cos(\sigma\pm\alpha)ds\frac{dw}{dw_1} \tag{5b}$$

Furthermore, terms $\alpha\epsilon$ (but not $\Delta\alpha$), $\epsilon\Delta\alpha$, α/M^2, and $\Delta\alpha/M^2$ will be neglected.

Introducing the variable $d\hat{s} = ds(dw/dw_1)$

$$\frac{d^2}{ds^2} = \frac{d^2}{d\hat{s}^2}\left(\frac{dw}{dw_1}\right)^2 + \frac{d}{d\hat{s}}\frac{d}{ds}\left(\frac{dw}{dw_1}\right) \tag{6}$$

$$\frac{d\sigma}{d\hat{s}} = \frac{d\sigma}{ds}\frac{dw_1}{dw} = K\frac{dw_1}{dw} \tag{7}$$

$$\frac{dy}{d\hat{s}} = \frac{dy}{ds}\frac{dw_1}{dw} = \sin\sigma\frac{dw_1}{dw} \tag{8}$$

Without an assumption about the streamlines, no expression can be developed for dw/dw_1. Assuming that their slopes are determined by the values immediately behind the shock, one gets $dw/dw_1 = 1 \mp (1/2y_1)\sin\alpha\, s_1$, where s_1 is the length of the meridian curve integrated from stagnation point to point of observation. Here one simply assumes that for small α it is of the form $dw/dw_1 = 1 \pm$ corr, where corr is proportional to α. Therefore, the second term in (6) can be neglected if it occurs with a factor ϵ.

Equation (3) now can be written as

$$\frac{dz}{d\hat{s}} = \epsilon\frac{qn}{qt}\frac{dw_1}{dw} + \frac{z}{\Delta}\left(\frac{d\Delta}{d\hat{s}} - \epsilon\frac{q_n}{q_t}\frac{dw_1}{dw}\right)$$

Introducing $d\Delta/d\hat{s}$ from Eq. (5a),

$$\frac{dz}{d\hat{s}} = \epsilon\frac{q_n}{q_t}\left[\frac{dw_1}{dw} + \frac{z}{\Delta}\left(1-\frac{dw_1}{dw}\right)\right] + z\left(\frac{1}{\epsilon}\frac{d\epsilon}{d\hat{s}} - \frac{1}{y_1}\frac{dy_1}{d\hat{s}}\right)$$

$$\frac{d^2z}{d\hat{s}^2} = \left[\frac{\epsilon}{\cos^2(\sigma\pm\alpha)}\frac{d\sigma}{d\hat{s}} + \tan(\sigma\pm\alpha)\frac{d\epsilon}{d\hat{s}} + \left(\frac{1}{\epsilon}\frac{d\epsilon}{d\hat{s}} - \frac{1}{y_1}\frac{dy_1}{d\hat{s}}\right)\epsilon\frac{q_n}{q_t}\right]\left[\frac{dw_1}{dw} + \frac{z}{\Delta}\left(1-\frac{dw_1}{dw}\right)\right] + z\left[-\frac{2}{\epsilon y_1}\frac{d\epsilon}{d\hat{s}}\frac{dy_1}{d\hat{s}} + \frac{1}{y_1^2}\left(\frac{dy_1}{d\hat{s}}\right)^2\right] + z\left[\frac{1}{\epsilon}\frac{d^2\epsilon}{d\hat{s}^2} + \frac{1}{y_1^2}\left(\frac{dy_1}{d\hat{s}}\right)^2 - \frac{1}{y_1}\frac{d^2y_1}{d\hat{s}^2}\right]$$

$$\frac{1}{\Delta}\int\frac{d^2z}{ds^2}dz = \left[\frac{\epsilon}{\cos^2(\sigma\pm\alpha)}K + 2\tan(\sigma\pm\alpha)\frac{d\epsilon}{ds} - \epsilon\tan(\sigma\pm\alpha)\frac{\sin\sigma}{y_1}\right]\left[1 + \frac{1}{2}\times\left(\frac{dw}{dw_1}-1\right)\right] - \frac{\Delta}{\epsilon y_1}\sin\sigma\frac{d\epsilon}{ds} + \Delta\frac{\sin^2\sigma}{y_1^2} + \frac{\Delta}{2\epsilon}\frac{d^2\epsilon}{ds^2} - \frac{\Delta}{2y_1}\cos\sigma K \tag{9}$$

The term $1 + \frac{1}{2}[(dw/dw_1) - 1]$ is multiplied by ϵ or $1/M^2$ because $d\epsilon/ds$ is proportional to $1/M^2$; it therefore may be replaced by 1. If the curvature of the shock is known, Eq. (9), together with Eq. (1), gives a very good representation of the pressure distribution at the body, as could be proved by using the shock shapes in Ref. 4. But in order to use this approach as an indirect method for the calculation of the body shape, one has to have a more accurate expression than Eq. (5a) for Δ. So far, no satisfactory expression could be found except for cones and spheres.

But Eq. (9) encourages an approach similar to that used in Ref. 3 for cones. Rotating the coordinate system from the shock axis to the wind axis by the angle α and introducing the coordinate $\hat{y} = (y/\sin\sigma)\sin(\sigma\pm\alpha)$ perpendicular to the wind, Eq. (9) becomes

$$\frac{1}{\Delta}\int\frac{d^2z}{ds^2}dz = \frac{\epsilon}{\cos^2(\sigma\pm\alpha)}K + 2\tan(\sigma\pm\alpha)\frac{d\epsilon}{ds} - \epsilon\tan(\sigma\pm\alpha)\frac{\sin(\sigma\pm\alpha)}{\hat{y}_1} - \frac{\Delta}{\epsilon\hat{y}_1}\sin(\sigma\pm\alpha)\frac{d\epsilon}{ds} + \frac{\Delta\sin^2(\sigma\pm\alpha)}{\hat{y}_1^2} + \frac{\Delta}{2\epsilon}\frac{d^2\epsilon}{ds^2} - \frac{\Delta\cos(\sigma\pm\alpha)K}{2\hat{y}_1} \tag{10}$$

wherein the last term $(\Delta/\hat{y}_1 \tan\sigma)$ was replaced by $\Delta/\hat{y}_1\tan(\sigma\pm\alpha)$, which is allowed within the neglections postulated below Eq. (5b).

The third and the fifth term in Eq. (10) are the only ones free of K and were shown in Ref. 3 to add up to $[-\epsilon\tan(\sigma\pm\alpha)/2y_1](dw_2/dw)$ for the cone. All the others are proportional to K because $d\epsilon/ds$ is proportional to K and $d^2\epsilon/ds^2$ depends on first and second curvature of the shock.

Received by IAS October 9, 1962.

* Scientist, Space Systems Division.

If one inserts Eq. (10) into Eq. (2) and then Eqs. (2) and (5b) into Eq. (1), one sees that the pressure is almost the pressure for an axisymmetric configuration with shock $\sigma \pm \alpha$, except that the curvature used in this pressure equation is no longer the curvature K of the original shock. The curvature of this equivalent axisymmetric shock is

$$\hat{K} = K(dw/dw_1) \tag{11}$$

It would have been surprising indeed if the shock for a general body of revolution at an angle of attack α_g would be of the same shape as the axisymmetric shock of this body turned by an angle $\eta = \alpha_g - \alpha$ (that is, at an angle α), as is the case for the cone. By coordinating the shock characterized by σ and K at angle of attack α to a body at angle α_g, a more complex relationship would be expected. The relationship indicated by Eq. (11) says that the curvature on the wind side is smaller by $1-$ corr (corr proportional to α) than the curvature K, and the curvature on the shadowside is correspondingly larger by $1+$ corr, but otherwise the equation for the pressure is the same as that for a shock $\sigma \pm \alpha$ in axisymmetric flow. If the shock of a sphere cone with cone half angle $\theta + \alpha_g$ is compared to that of a sphere cone with cone half angle θ, both in axisymmetric flow, it is seen that the curvature of the larger sphere cone is indeed smaller.

The result of the present investigation is not a solution of the problem but a suggestion to replace the problem by an axisymmetric one with a body $\theta \pm \alpha_e$. The whole investigation is restricted to small α, and, if the afterbody is conical, it is suggested that the conical relation between the effective and the geometrical angles of attack be used [see Eq. (8) in Ref. 3].

Comparison of measurements for sphere cones at angle of attack with such measurements for equivalent sphere cones at zero angle of attack shows excellent agreement with each other and with numerical calculations according to Eqs. (1–11), if based on the axisymmetric shock of the equivalent body.

References

[1] Hayes, W., "Some aspects of hypersonic flow," Ramo-Wooldridge Rept. (1955).

[2] Hayes, W. and Probstein, R., *Hypersonic Flow Theory* (Academic Press Inc., New York, 1959), pp. 119–129, 139–165.

[3] Ginzel, I., "Two remarks on cones at angle of attack in high supersonic flow," J. Aerospace Sci. **29**, 497–498 (1962).

[4] Kubota, T., "Investigations of flow around simple bodies in hypersonic flow," Guggenheim Aeronaut. Lab., Calif. Inst. Tech. Hypersonic Research Memo. 40 (1957).

Approximate Determination of the Incompressible Flow Region in Front of a Blunt Body in Hypersonic Flow

M. A. Rahman*
Douglas Aircraft Company, Santa Monica, Calif.

Nomenclature

M = Mach number
μ = strength of doublet
K = ratio of specific heats
R = gas constant
T = absolute temperature

Received by IAS October 15, 1962.

* Associate Engineer, Missiles and Space Division.

Subscripts

1 = ahead of shock
2 = behind shock

In the case of hypersonic flow, a shock wave stands in front of the blunt body and forms a region of nearly incompressible flow around the stagnation region; the flow, behind the shock and in the vicinity of the x axis (see Fig. 1), may be assumed to be nearly uniform. In this paper an attempt is made to determine the extent of the incompressible flow region in front of a hemispherical body (two-dimensional case). The shock-detachment distance δ^{*1} is assumed to be known.

In the field of hydrodynamics,[2] popular use is made of the method of combining two flow patterns (namely, the source in a uniform flow) and of interpreting the results as the flow past a rigid body. Here a similar approach is adapted to find the flow characteristics on the cylindrical curvature of the body. Therefore, a uniform flow is superimposed on a doublet at O (see Fig. 1), and either the potential function or the stream function can be used for the solution. The author's preference is to use the stream function here.

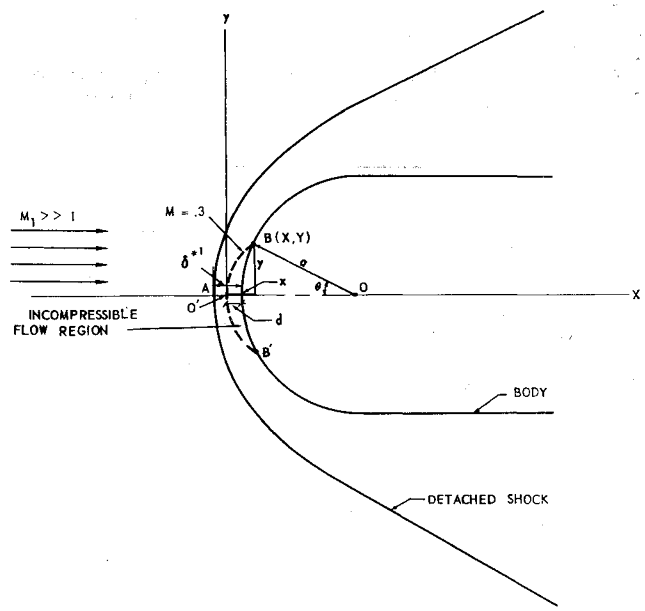

Fig. 1 Axially symmetric flow

The stream function for the combination of a doublet and a uniform flow with velocity u_2 in the positive x direction is

$$\psi = u_2 y - \mu/2\pi r^2 \cdot y \tag{1}$$

Writing $\mu = 2\pi a^2 u_2$, the stream function becomes

$$\psi = u_2 y[1 - (a^2/r^2)] = u_2[r - (a^2/r)] \sin\theta \tag{2}$$

The velocity at any point is expressed most conveniently in polar coordinates, and the radial and circumferential components are, respectively,

$$u_r = (1/r)(\partial\psi/\partial\theta) = u_2[1 - (a^2/r^2)] \cos\theta \tag{3}$$

$$u_\theta = -\partial\psi/\partial r = -u_2[1 + (a^2/r^2)] \sin\theta \tag{4}$$

On the cylindrical surface $r = a$, u_r vanishes, and the circumferential component becomes

$$u_\theta = -2u_2 \sin\theta \tag{5}$$

The equation of a circle with its center at point (r,O') and radius r is

$$(x - r)^2 + y^2 = r^2 \tag{6}$$

Now, relating the shock detachment distance δ^* with the dis-